

Function Whose Derived Function is Equal to it's Inverse Function

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1 Introduction

To date, few examples of a function whose derived function equal to it's inverse function have been discovered. I want to discover and study these cases to determine its rules.

2-1 Methods

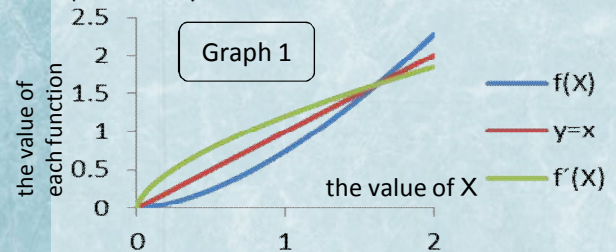
Plan $f(x) =$ (literal expression)
 ↓
 Calculated derived function and inverse function.

$$f'(x), f^{-1}(x)$$

solved $f'(x) = f^{-1}(x)$ as identical equation of x .

Practice

For example, when $f(x)$ expresses $ax^n (x \geq 0, f(x) \geq 0)$, a is n^{1-n} , n is $(1+5^{1/2})/2$. (Graph 1)



It can be found that $f(x)$ and $f'(x)$ has line symmetry to $x=y$.

It is difficult to solve $f'(x) = f^{-1}(x)$ as identical equation of x !

2-2 Methods

Isn't there any another way to find $f(x)$?

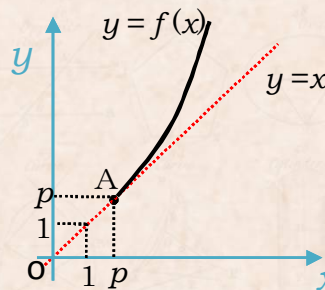
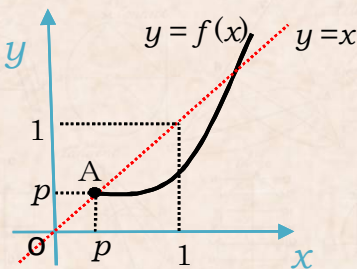


Think and find $f(x)$'s feature.

※No graph 1

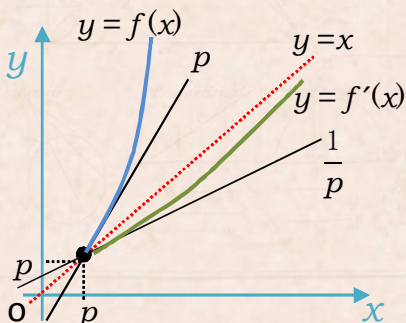
Feature

- ① $f(x)$ passes Point P (p, p) .
- ② When $f(x)$ is in first quadrant, $f(x)$'s domain is $x \geq p$.
- ③ Count the number of the intersection point of $f(x)$ and straight line $x=y$. There are two possibilities: two points in $0 \leq p < 1$ and one point in $1 \leq p$.



Consideration

Considering all possibilities, it is realized that $f(p)=p, f'(p)=p$ and $f''(p)=1/p$.



Future deployment

solving the system of equations there.
 $f(p)=p, f'(p)=p,$
 $f''(p)=\frac{1}{p}$

3 Conclusions

I found characteristics of $f(x)$ and a new method to find $f(x)$ in this study.

I will try to test new method in the future.